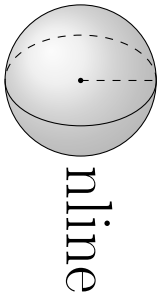
cyber math



pen Tournament

nline

Sample Problems

Saturday, March 26, 2022

These are some sample problems that are meant to represent the style and general difficulty level of the problems featured in the upcoming Open Online Tournament!

High School Division

1. What is the remainder when $1^{2020} + 2^{2020} + 3^{2020} + \cdots + 2020^{2020}$ is divided by 2021?

2. Suppose that
$$\sum_{k=1}^{\infty}(2k+1)r^k = 3r + 5r^2 + 7r^3 + 9r^4 + \cdots = \frac{27}{25}$$
for some real number $-1 < r < 1$. The value of r can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

3. Suppose that x and y are randomly chosen integers from 1 to 5, inclusive, with $x < y$. The expected value of $x^2 + 2y^2$ can be written in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. Compute $p + q$.

4. A positive integer is called *timid* if any two consecutive digits in the integer are either equal or only 1 apart. For example, neither 469 nor 2020 is timid, but 2123 and 4456 are. Compute the number of timid four-digit integers.

5. What is the sum of all prime numbers p such that $p+2$ is prime that satisfy the congruence $p^p + p \equiv 1 \pmod{p+2}$?

6. What is the total volume of the figure in the 3-dimensional coordinate plane containing all points that are within 1 unit of some point lying on a line segment with length 1?

7. What is the remainder when 5^{97} is divided by 23?

8. Among all permutations of the word TOURNAMENT, compute the number of times in which the substring TN appears in total.

9. What is the remainder when $123456 \cdots 202020212022$ is divided by 18?

10. Let r and s be the roots of $x^2 + 7x + 11$. Compute $\sqrt{r} + \sqrt{s}$.